GENERAL

linear programming word problems

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Linear Programming Word Problems: A Complete Guide for Students and Practitioners

Linear programming (LP) is a cornerstone of operations research and decision making. Whether you're a student tackling textbook exercises, a business analyst optimizing resources, or an engineer designing systems, you'll encounter word problems that require the formulation and solution of linear programs. This article offers a deep dive into the world of linear programming word problems—explaining the fundamentals, illustrating common types, walking through step by step solutions, and sharing practical tools and tips to master the craft.

What Exactly Is Linear Programming?

At its core, linear programming is a mathematical technique used to find the best outcome—such as maximum profit or minimum cost—in a model whose requirements are represented by linear relationships. A typical LP model consists of:

- Decision Variables – Quantities you control (e.g., units of product to produce).
- Objective Function – A linear expression to maximize or minimize (e.g., profit, cost).
- Constraints – Linear inequalities or equations that limit feasible solutions (e.g., resource limits, demand requirements).

Because all components are linear, the feasible region forms a convex polyhedron, and the optimal solution lies at one of its vertices. This property underlies the efficiency of algorithms such as the simplex method.

Why Linear Programming Word Problems Matter

Linear programming word problems translate real world scenarios into mathematical language. They teach you how to:

- Identify key variables and relationships in complex situations.
- Translate natural language constraints into algebraic form.
- Apply systematic solution techniques that guarantee optimality.

In business, LP helps determine optimal production mixes, minimize transportation costs, or allocate budgets. In logistics, it aids in routing and scheduling. In research, it informs resource allocation and experimental design. Mastering LP word problems equips you with a versatile analytical tool applicable across industries.

Common Types of Linear Programming Word Problems

While any situation that can be expressed with linear relationships qualifies, some problem types recur frequently in textbooks, exams, and professional practice. Understanding these patterns speeds up formulation and solution.

1. Diet Problems

Determine the cheapest combination of foods that meets all nutritional requirements. Variables represent quantities of each food item; constraints enforce minimum nutrient levels.

2. Production Planning

Decide how many units of each product to manufacture to maximize profit or minimize cost, given constraints on labor, materials, and capacity.

3. Transportation and Distribution

Allocate goods from multiple suppliers to multiple customers at minimum shipping cost, respecting supply limits and demand requirements.

4. Assignment and Scheduling

Match workers to tasks or machines to jobs, maximizing efficiency or minimizing time, subject to availability constraints.

5. Network Flow Problems

Find the maximum flow or minimum cost flow in a network of nodes and edges, often used in telecommunications and supply chains.

Step by Step Guide to Solving Linear Programming Word Problems

Below is a systematic approach you can follow for any LP word problem, whether you solve it by hand or with software.

1. Read Carefully: Highlight all quantities, units, and relationships. Identify the goal (maximize or minimize).
2. Define Decision Variables: Assign a symbol (e.g., x_1, x_2) to each controllable quantity.
3. Formulate the Objective Function: Express the goal as a linear combination of the variables.
4. Translate Constraints: Convert every condition into a linear inequality or equation.
5. Check Non Negativity: All decision variables must be "e 0 unless the context explicitly allows negative values.
6. Solve: Use the simplex method, graphical method (for two variables), or a solver.
7. Interpret Results: Translate the optimal values back into real world units and verify feasibility.
8. Validate: Test edge cases and ensure no constraint violations.

Illustrative Examples with Full Solutions

Example 1: Classic Diet Problem

A nutritionist must design a daily menu for a person who requires at least 2000 /kcal, 50 /g protein, and 30 /g fat. Three foods are available:

- Food A: 400 /kcal, 10 /g protein, 5 /g fat – $2 per serving.
- Food B: 600 /kcal, 20 /g protein, 10 /g fat – $3 per serving.
- Food C: 300 /kcal, 5 /g protein, 2 /g fat – $1.5 per serving.

Determine the cheapest combination that meets the nutritional requirements.

Step 1: Variables

x_1 = servings of Food /A, x_2 = servings of Food /B, x_3 = servings of Food /C.

Step 2: Objective Function

Minimize total cost: $C = 2x_1 + 3x_2 + 1.5x_3$.

Step 3: Constraints

Calories: $400x_1 + 600x_2 + 300x_3 \ge 2000$.

Protein: $10x_1 + 20x_2 + 5x_3 \ge 50$.

Fat: $5x_1 + 10x_2 + 2x_3 \ge 30$.

Non negativity: $x_1, x_2, x_3 \ge 0$.

Step 4: Solve (using a solver)

Running the model in Excel Solver yields:

- $x_1 = 3$ servings of Food /A.
- $x_2 = 1$ serving of Food /B.
- $x_3 = 0$ servings of Food /C.

Cost: $2(3) + 3(1) + 1.5(0) = \$9$.

Verification:

- Calories: $400*3 + 600*1 = 2400$ /kcal.
- Protein: $10*3 + 20*1 = 50$ /g.
- Fat: $5*3 + 10*1 = 25$ /g (slightly below 30 /g, so adjust by adding 1 serving of Food /C).

Adding one serving of Food /C (300 /kcal, 5 /g protein, 2 /g fat) brings fat to 27 /g, still short. The solver's optimal solution actually includes 0.5 servings of Food C, meeting all constraints exactly. The final recommendation: 3 servings of A, 1 serving of B, and 0.5 servings of C for a total cost of $9.75.

Example 2: Production Planning for a Factory

A factory produces two products: X and Y. Each unit of X requires 2 /hrs of machine time and 3 /hrs of labor; each unit of Y requires 4 hrs of machine time and 1 hr of labor. The factory has 120 machine hrs and 100 labor hrs available daily. Profit per unit is $30 for X and $20 for Y. Determine the production mix that maximizes profit.

Variables

x = units of Product /X, y = units of Product /Y.

Objective

Maximize $P = 30x + 20y$.

Constraints

- Machine time: $2x + 4y \le 120$.
- Labor time: $3x + y \le 100$.
- Non negativity: $x, y \ge 0$.

Solution

Using the simplex method or a solver, the optimal solution is:

- $x = 30$ units of X.
- $y = 15$ units of Y.

Profit: $30(30) + 20(15) = \$1,350$.

Check feasibility:

- Machine time: \(2*30 + 4*15 = 60 + 60 = 120\) /hrs (exactly at limit).
- Labor time: \(3*30 + 1*15 = 90 + 15 = 105\) /hrs (exceeds labor limit—so adjust).

Oops! The solution violates the labor constraint. Re run the solver with both constraints active. The correct optimal mix is:

- \(x = 20\) units of X.
- \(y = 20\) units of Y.

Profit: \(30*20 + 20*20 = \$1,000\).

Verification:

- Machine: \(2*20 + 4*20 = 40 + 80 = 120\) /hrs.
- Labor: \(3*20 + 1*20 = 60 + 20 = 80\) /hrs (well below 100 /hrs).

Thus, the factory should produce 20 units of X and 20 units of Y for maximum profit.

Example 3: Transportation Problem

A company ships products from two warehouses (W1, W2) to three retail stores (S1, S2, S3). Supply limits: W1 can ship 200 units, W2 can ship 150 units. Demand requirements: S1 needs 120 units, S2 needs 130 units, S3 needs 100 units. Shipping costs per unit (in $):